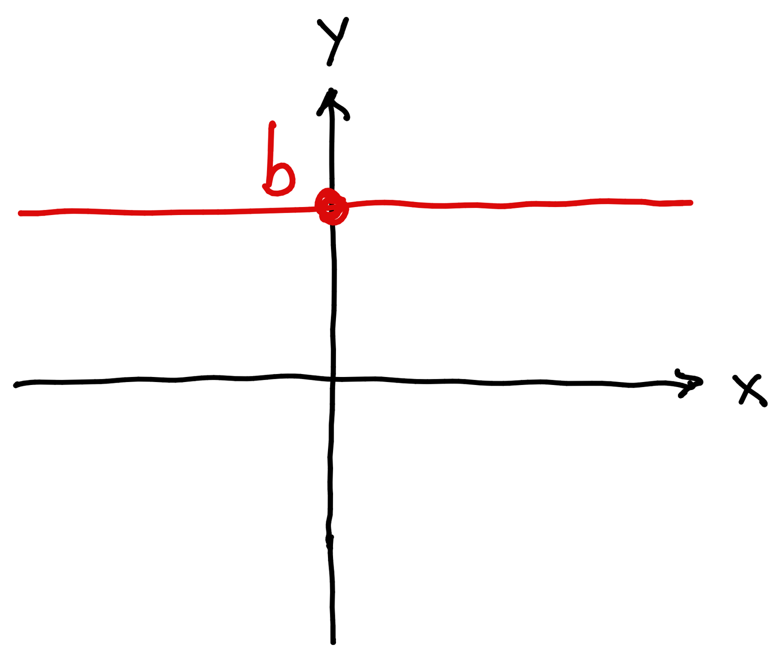
Class 11 – Transformations of Functions

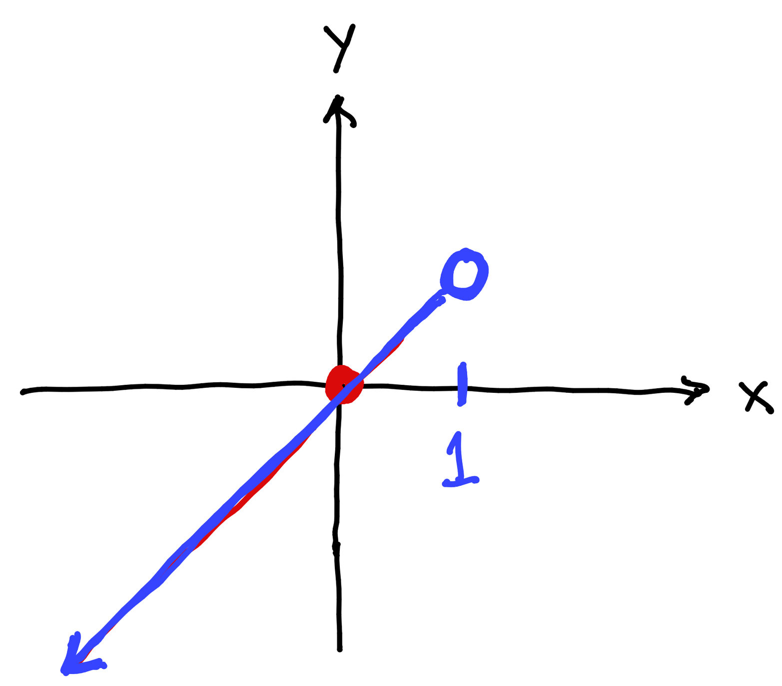
Review:

constant $f(x)=b$



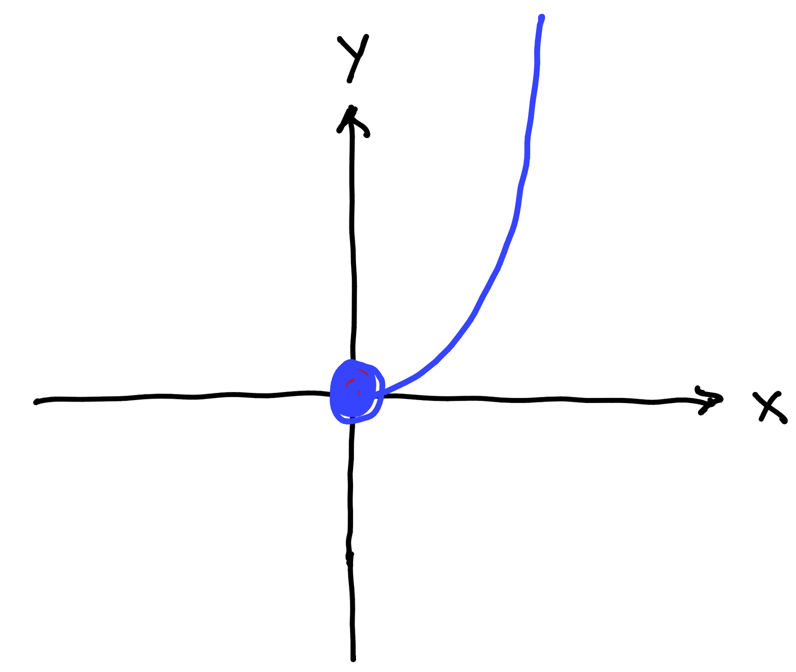
$x<1$

indentity $f(x)=x$

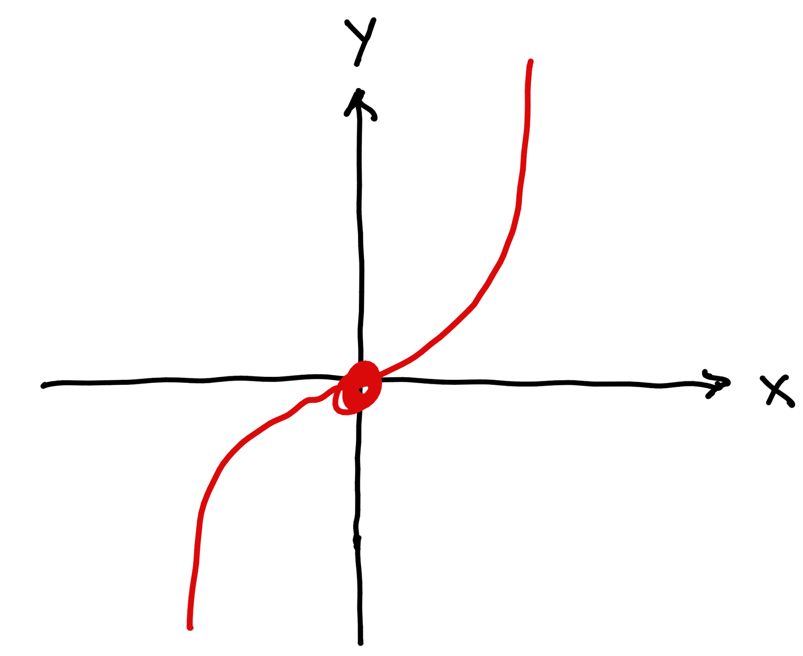


$x\geq 0$

square $f(x)=x^2$

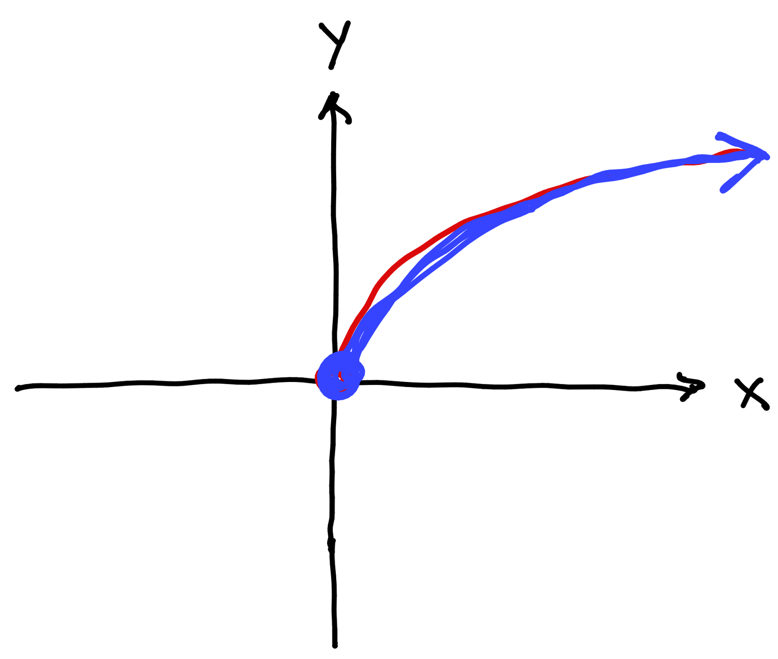


cubic $f(x)=x^3$



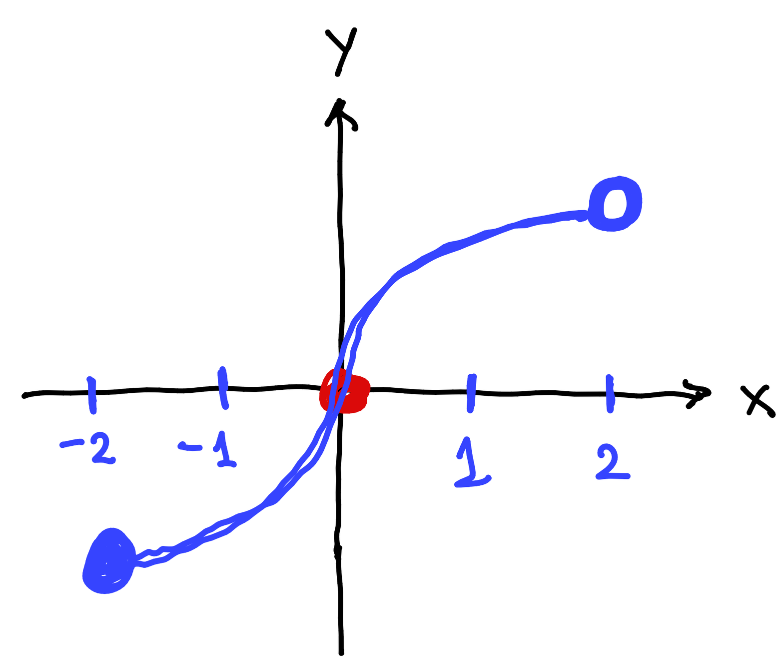
$x\geq -3 \leftrightarrow [-3,\infty)$

square root $f(x)=\sqrt{x}$

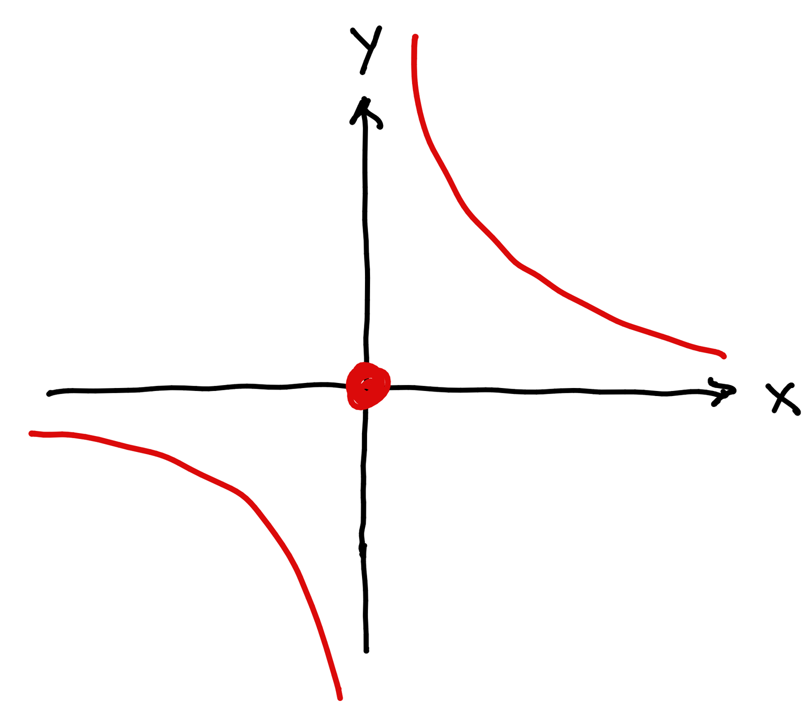


$-2\leq x<2$

cubic root $f(x)=\sqrt[3]{x}$

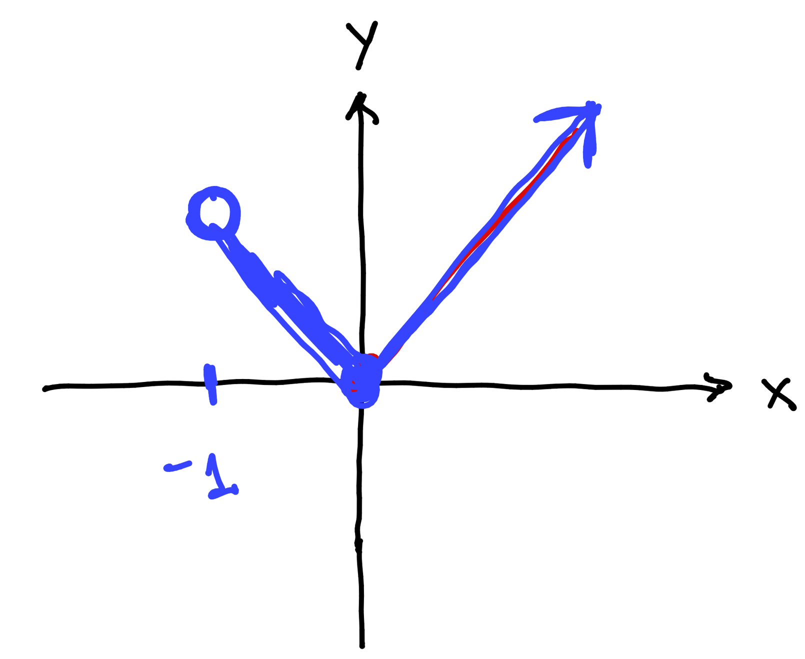


reciprocal $f(x)=\frac{1}{x}$



$x>-1$

Abs. value $f(x)=|x|$



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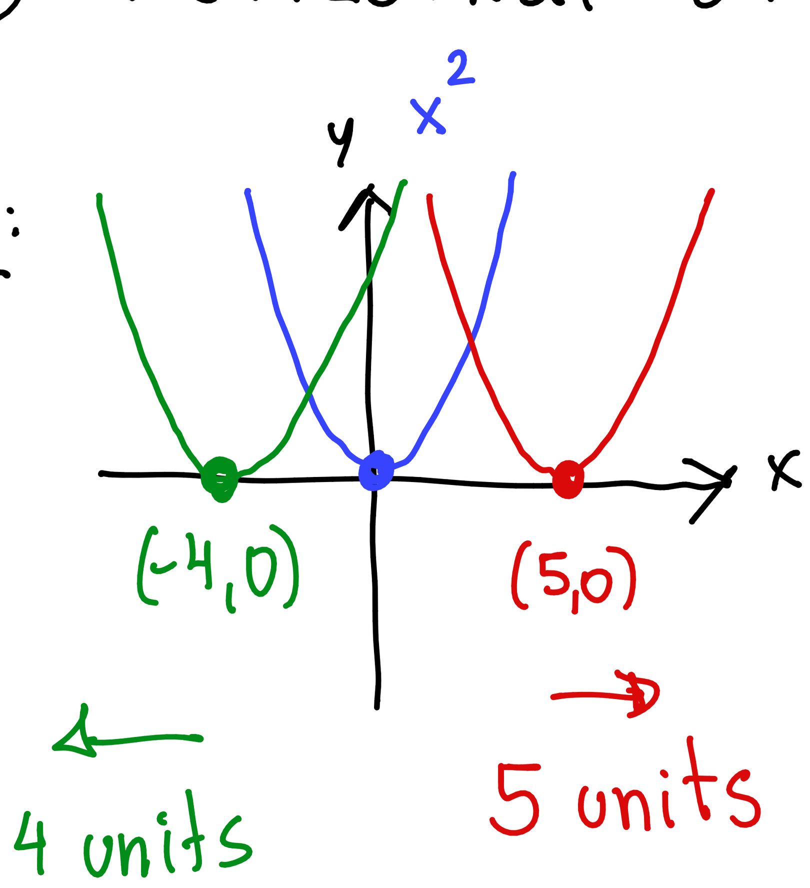
① Horizontal shift : subtract a shift to the right with the variable $\underline{\underline{x}}$.

Ex1:

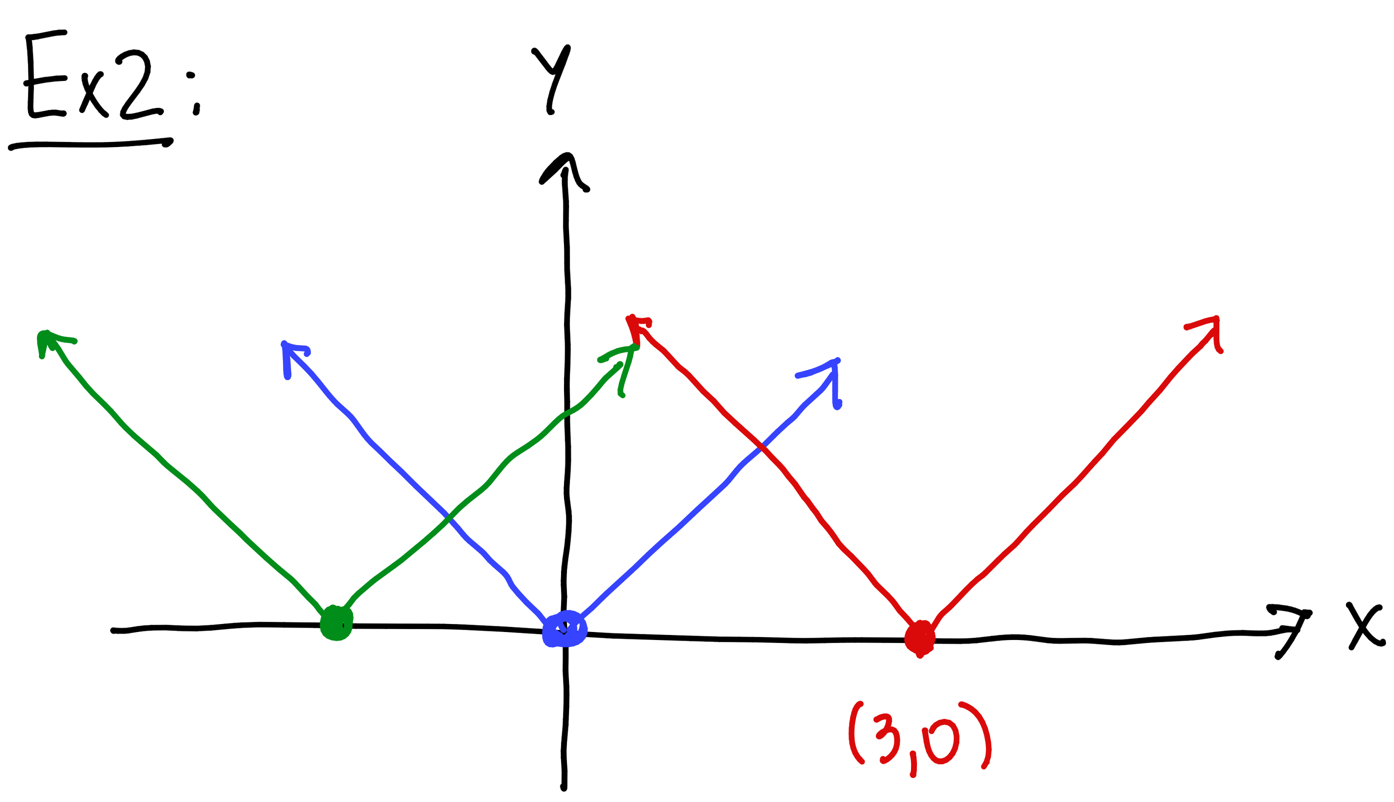


$f(x)=x^2$

$g(x)=(x-5)^2$, $g(5)=(5-5)^2=0$

$h(x)=(x+4)^2$

Ex2:



$f(x)=|x|$

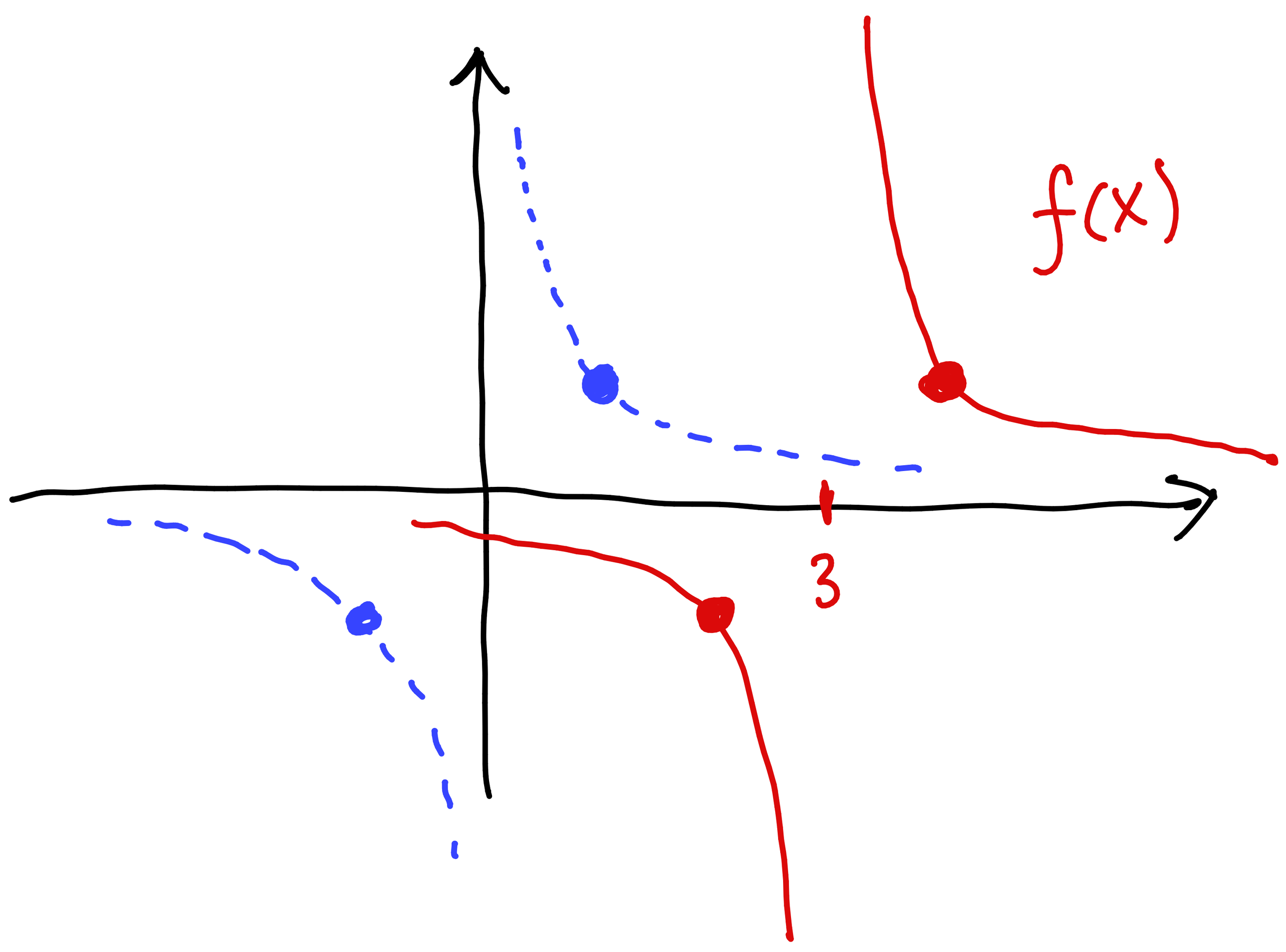
$g(x)=|x-3|$ 3 units →

$h(x)=|x+2|$ 2 units ←

Exercise: graph the function $f(x) = \frac{1}{x-3}$.

① identify basic function : $\frac{1}{x}$

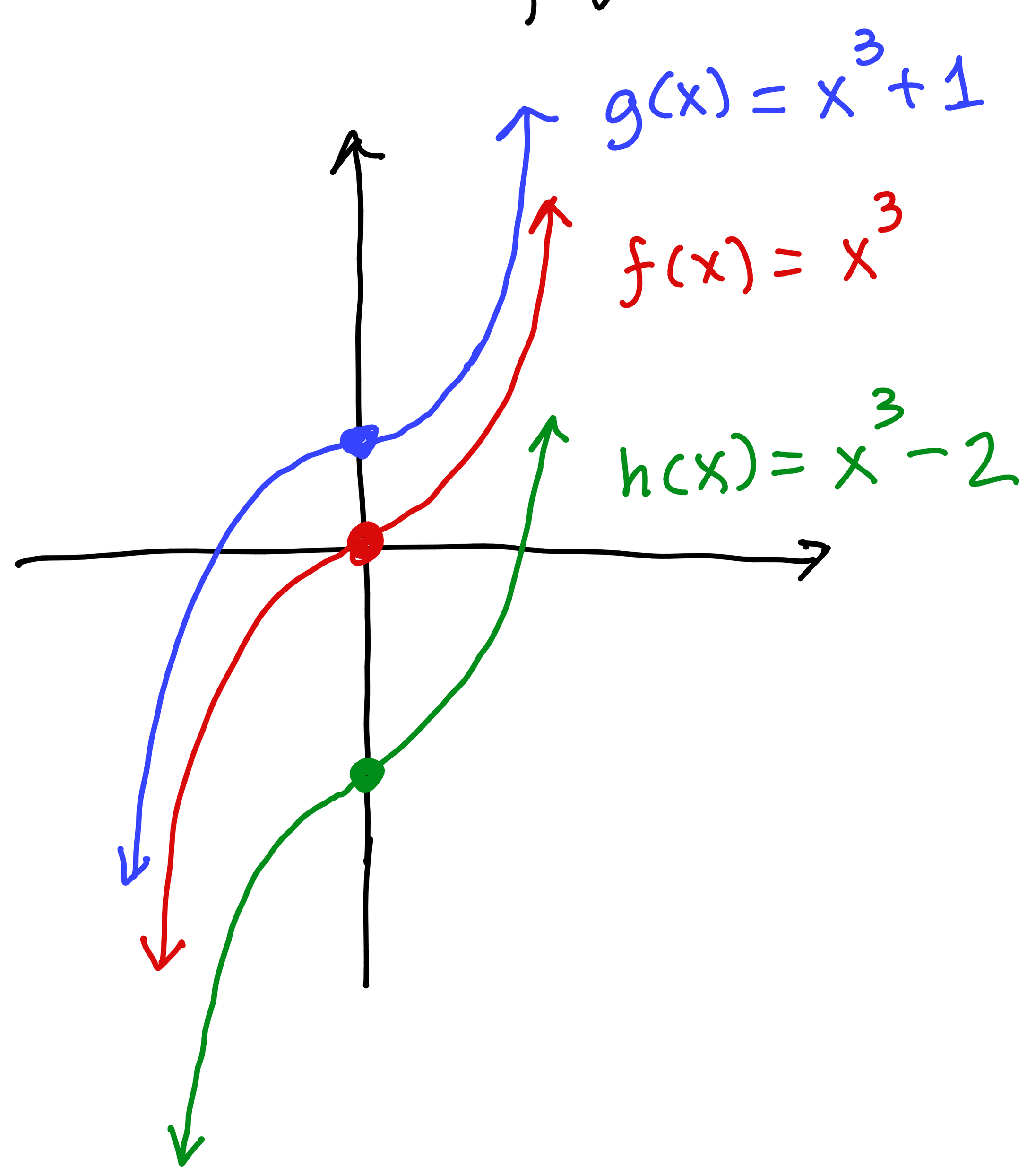
② identify the transformation : -3 or shifting 3 units →



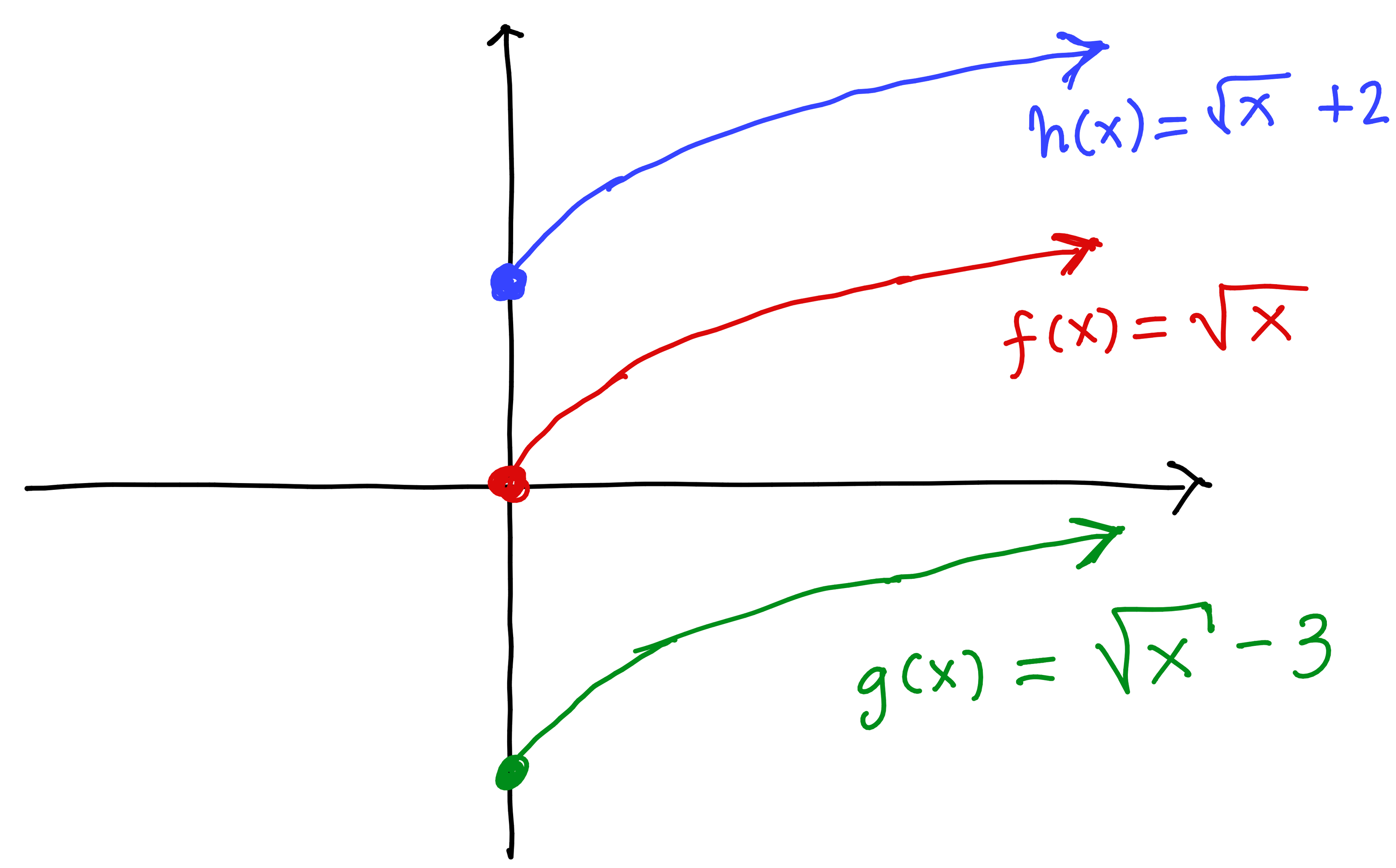
$\frac{6}{x} \to (\sqrt{6}, \sqrt{6})$ & $(-\sqrt{6}, -\sqrt{6})$

② Vertical shifts : add a shift at the "end" of a function.

Ex 1:



Ex 2:



↓ 2 units : -2 "at the end"

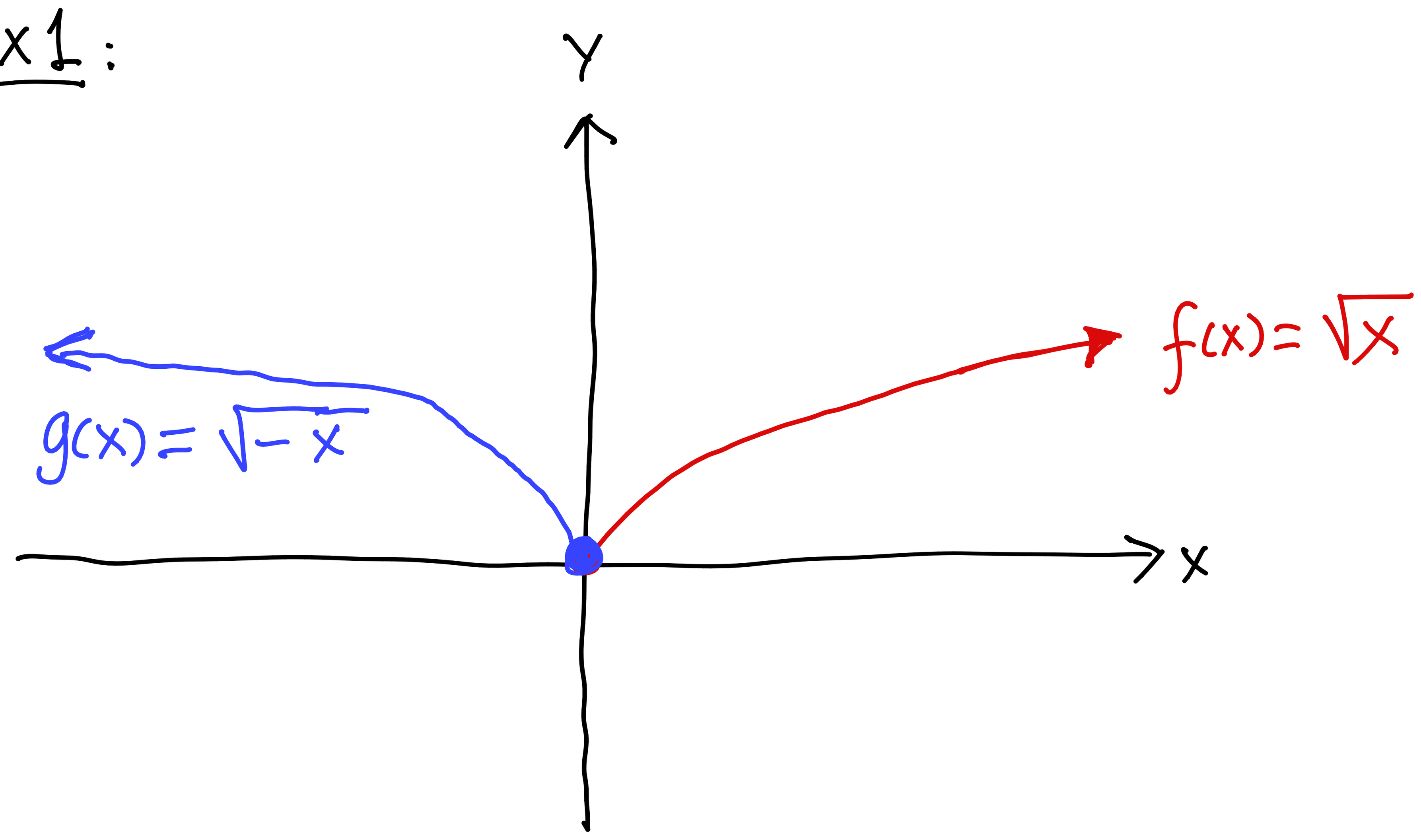
↑ 1 unit : $+1$ "at the end"

↓ 3 units : -3

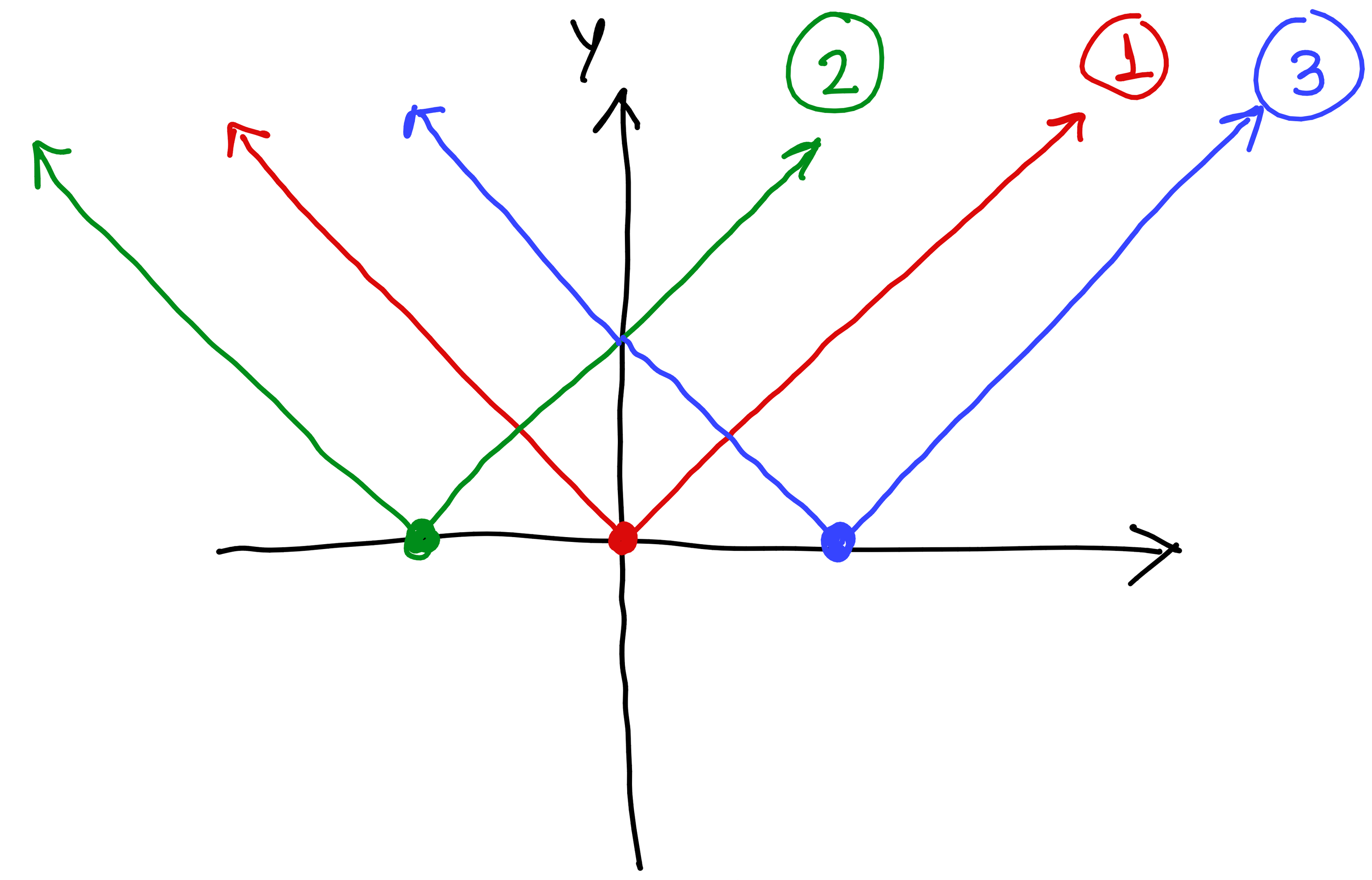
↑ 2 units : $+2$

③ Reflection about the y-axis: $f(-x)$, before the function operation.

Ex1:



Ex2: Graph $f(x)=|2-x|$



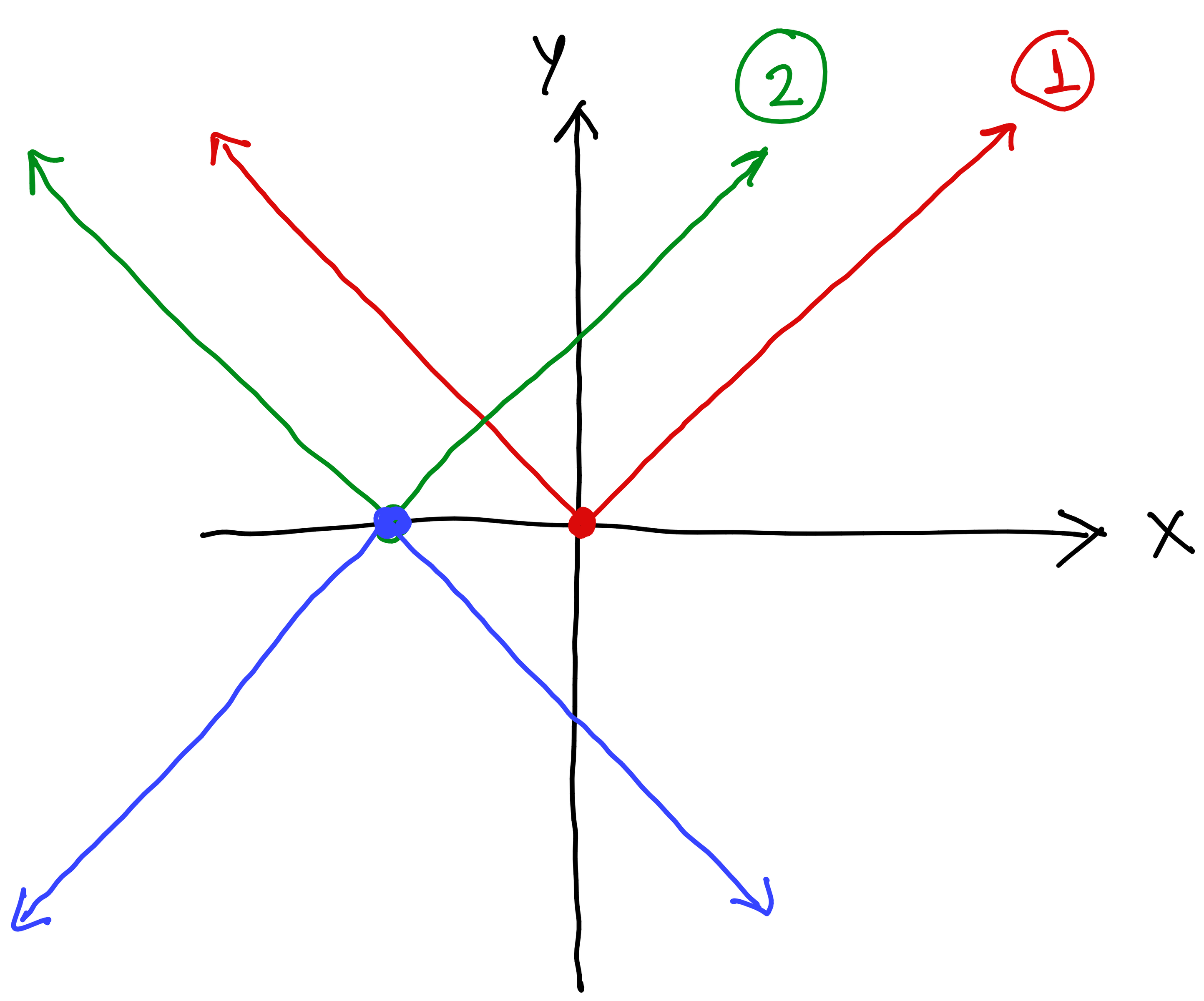
① basic function $|x|$

② +2 before the function: 2 ←

③ Consider $-x$: y- reflection

④ Reflection around the x-axis: $-f(x)$, after the function operation

Ex1: $f(x)=-|x+2|$

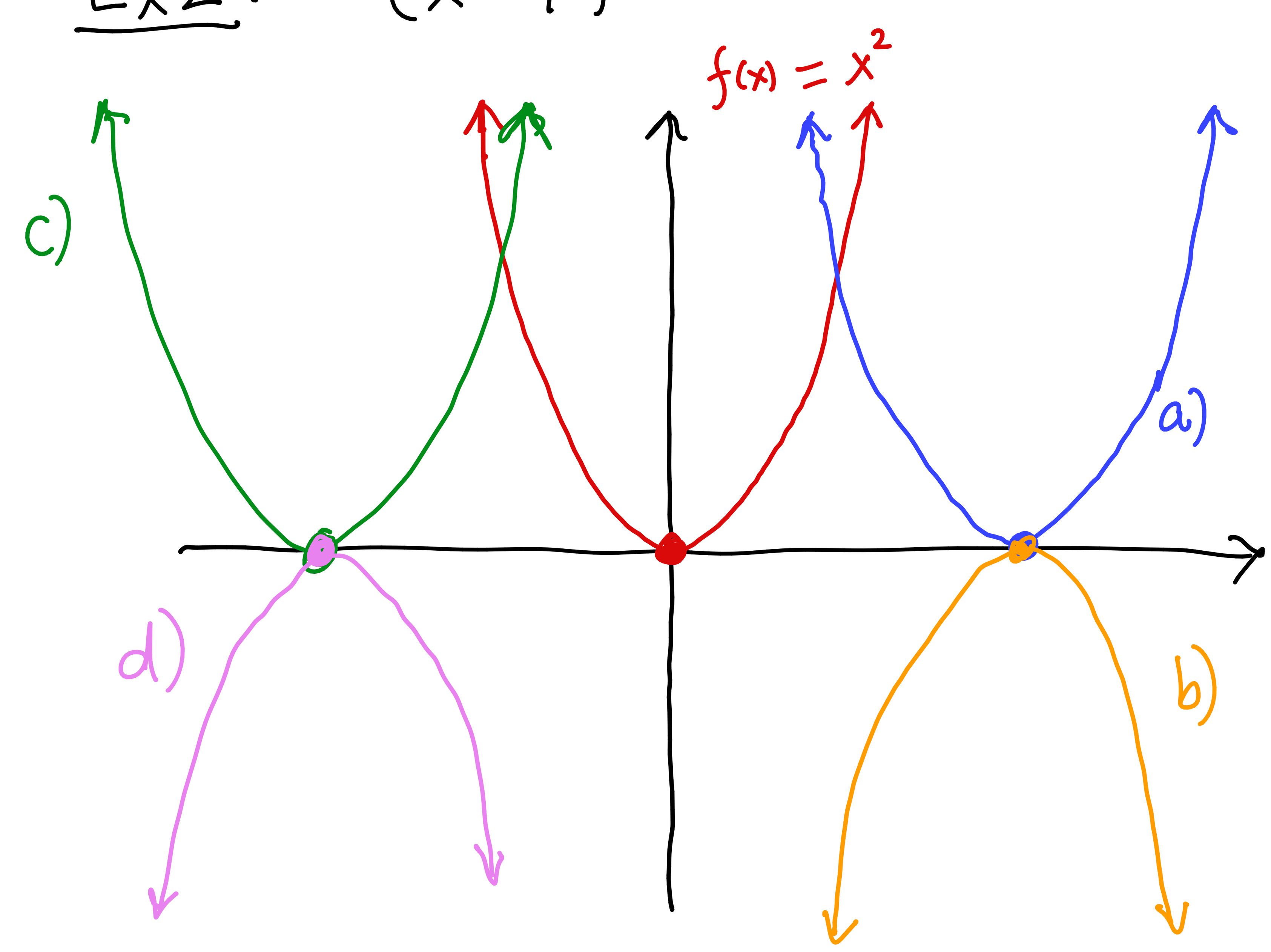


① basic function $|x|$

② +2 before the function: 2 ← $|x+2|$

③ Consider $-f(x)$. x-reflection $-|x+2|$

Ex2: $-(x-4)^2$



a) $g(x)$ $(x-4)^2$

b) $h(x)$ $-(x-4)^2$

c) $p(x)$ $(x+4)^2$

d) $q(x)$ $-(x+4)^2$

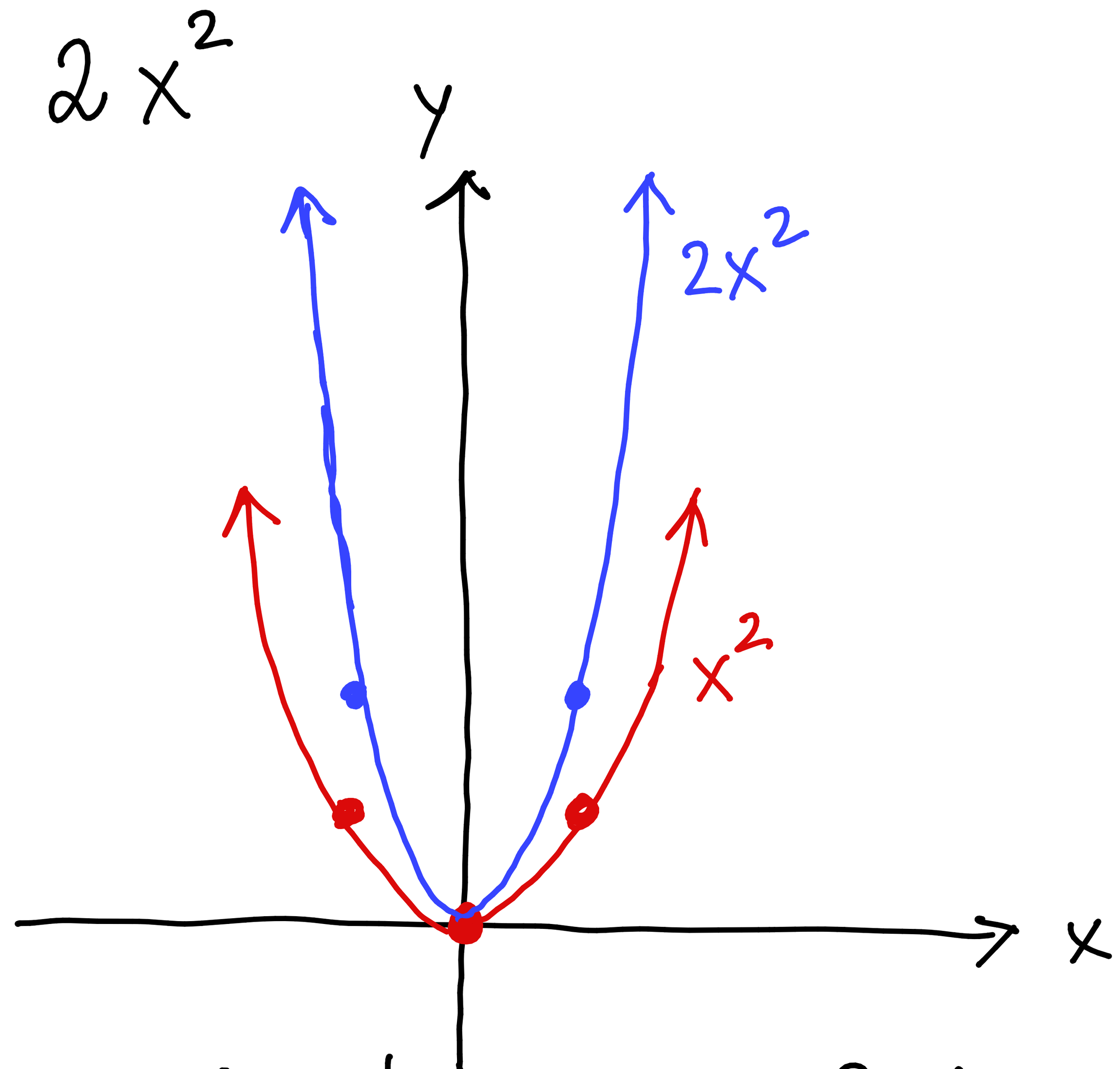
① basic function x^2

② horizontal shift : 4 →

③ x-reflection

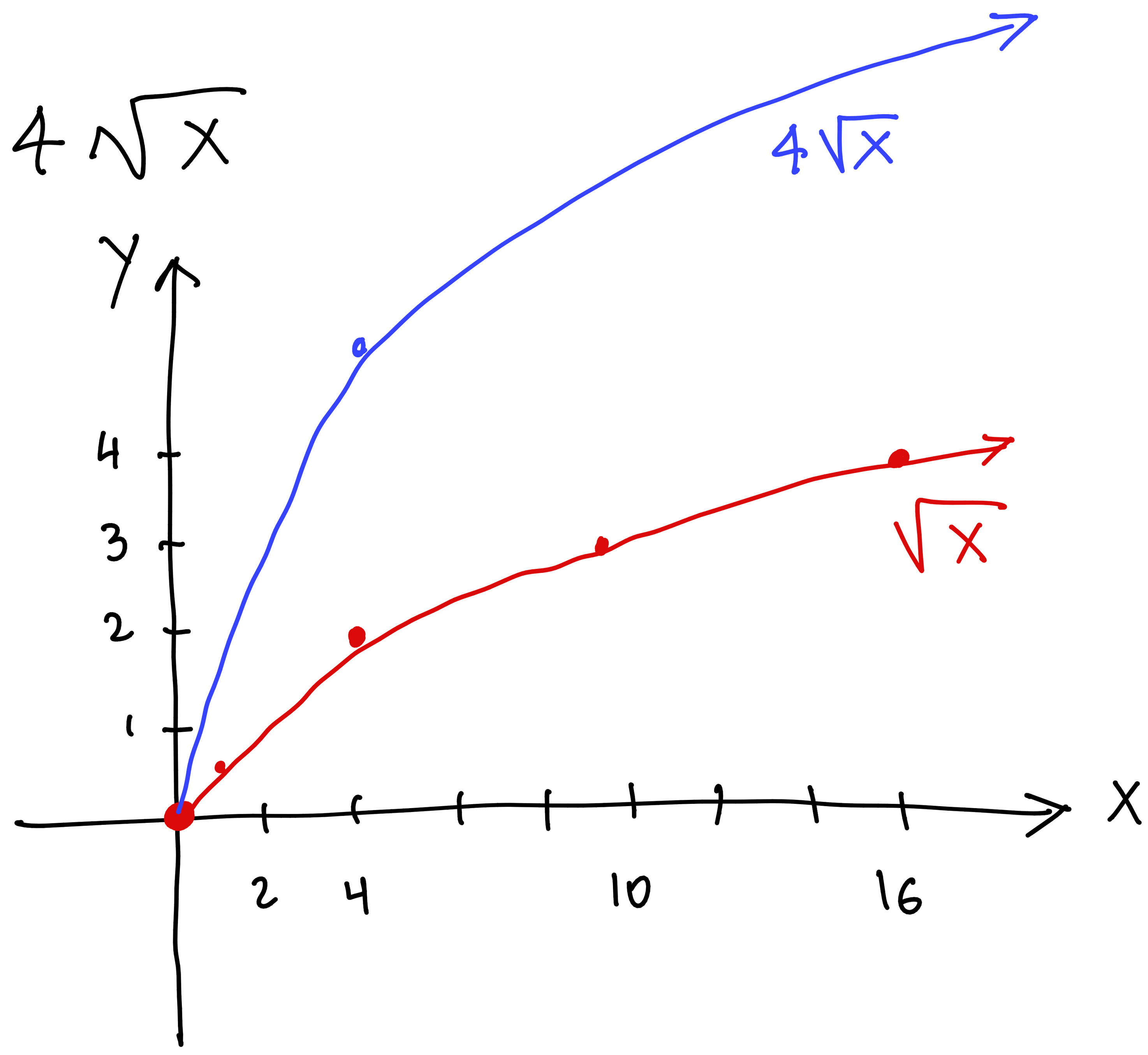
⑤ Stretching : $a \cdot f(x)$, where $a > 1$

Ex1: $2x^2$



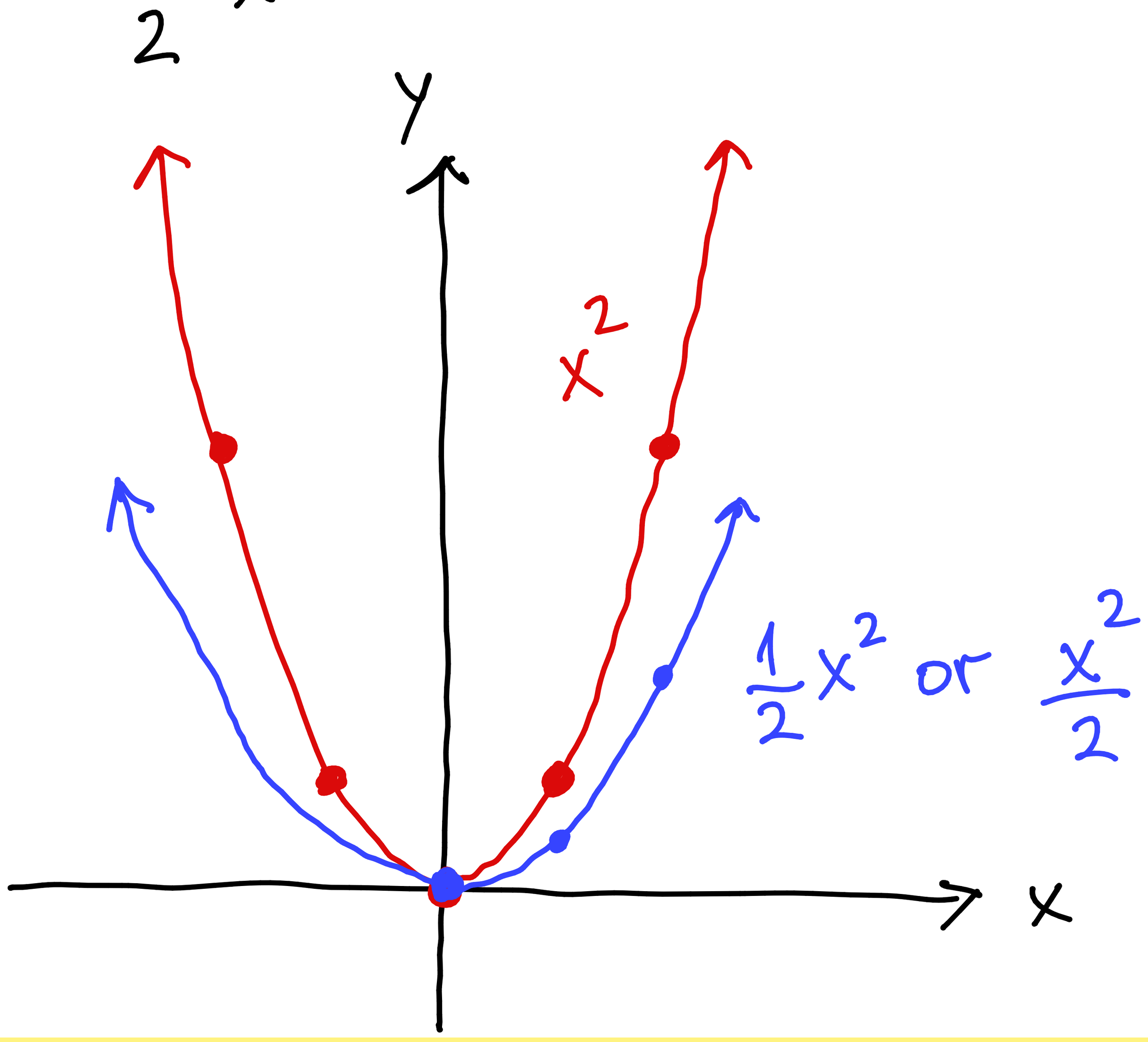
the basic function goes further away from the x-axis.

Ex2: $4\sqrt{x}$



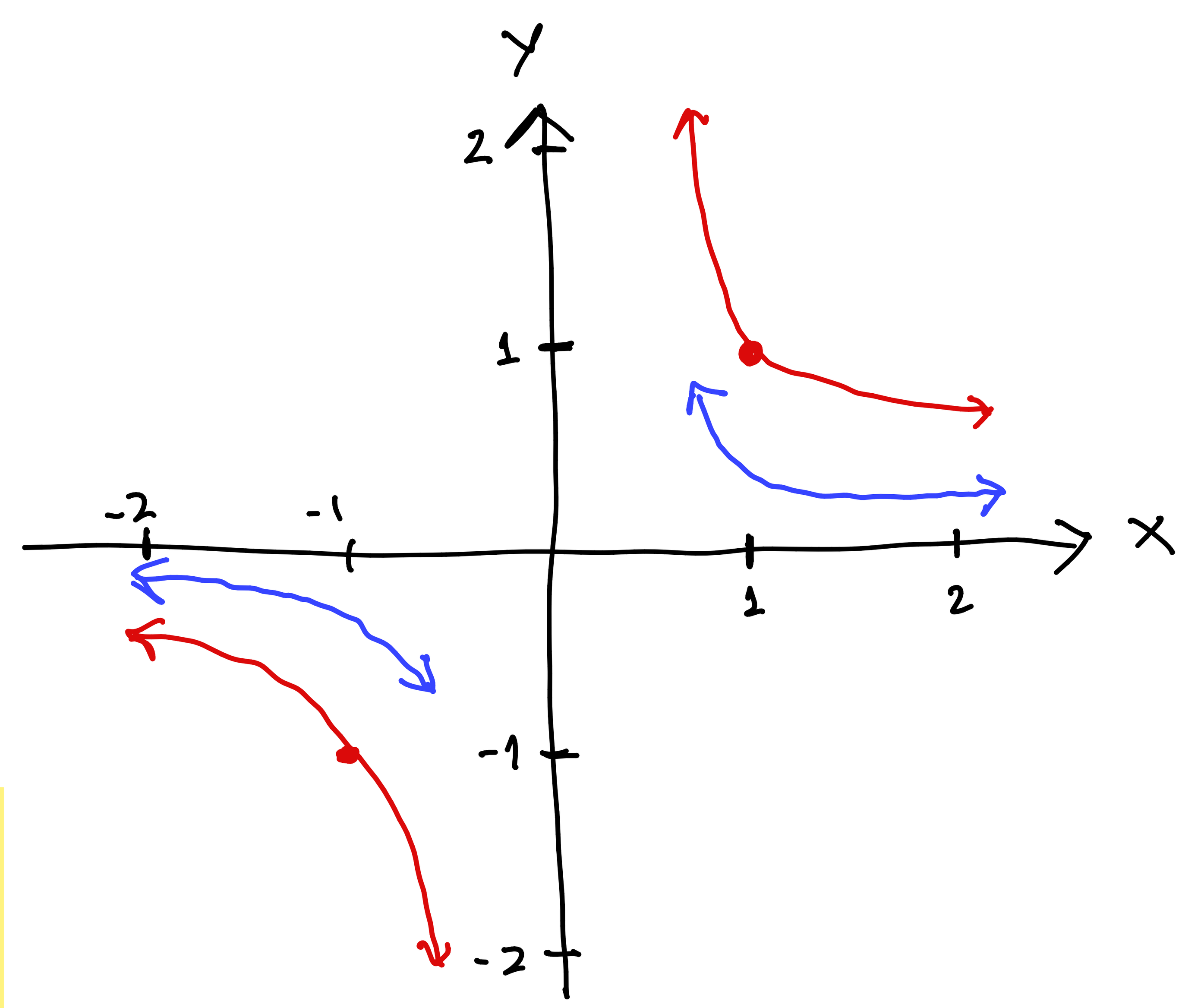
⑥ Compression: $a \cdot f(x)$, where $0 < a < 1$

Ex1: $\frac{1}{2}x^2$

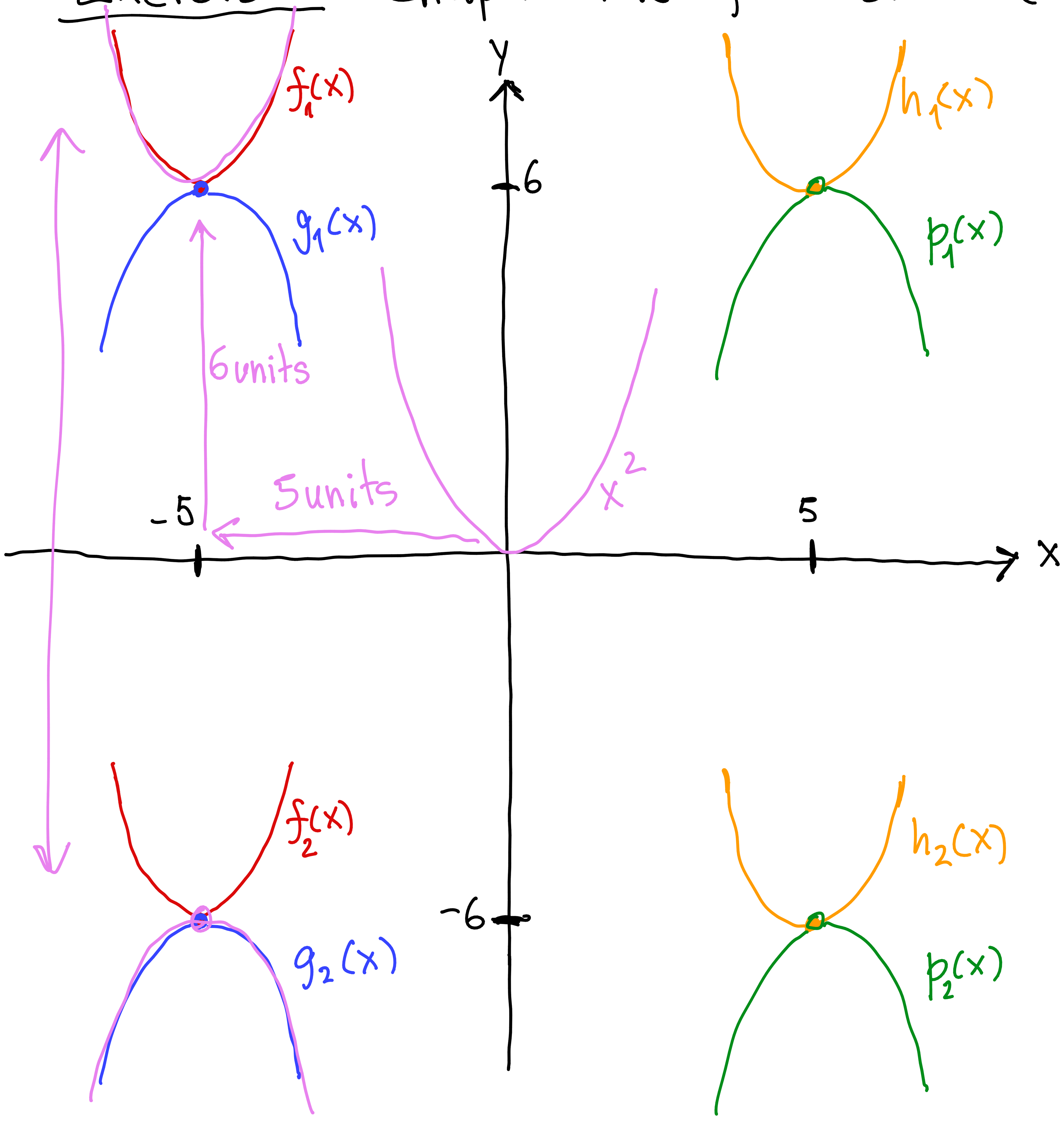


Remark: a compression makes the basic function go closer to the x-axis.

Ex2: $\frac{1}{5x} = \frac{1}{5} \cdot \frac{1}{x}$



Exercise: Graph the function $-(x+5)^2-6$.



a) $f_1(x)$

b) $g_1(x)$

c) $h_1(x)$

d) $p_1(x)$

e) $f_2(x)$

f) $g_2(x)$

g) $h_2(x)$

h) $p_2(x)$

Step 1: identify basic function x^2

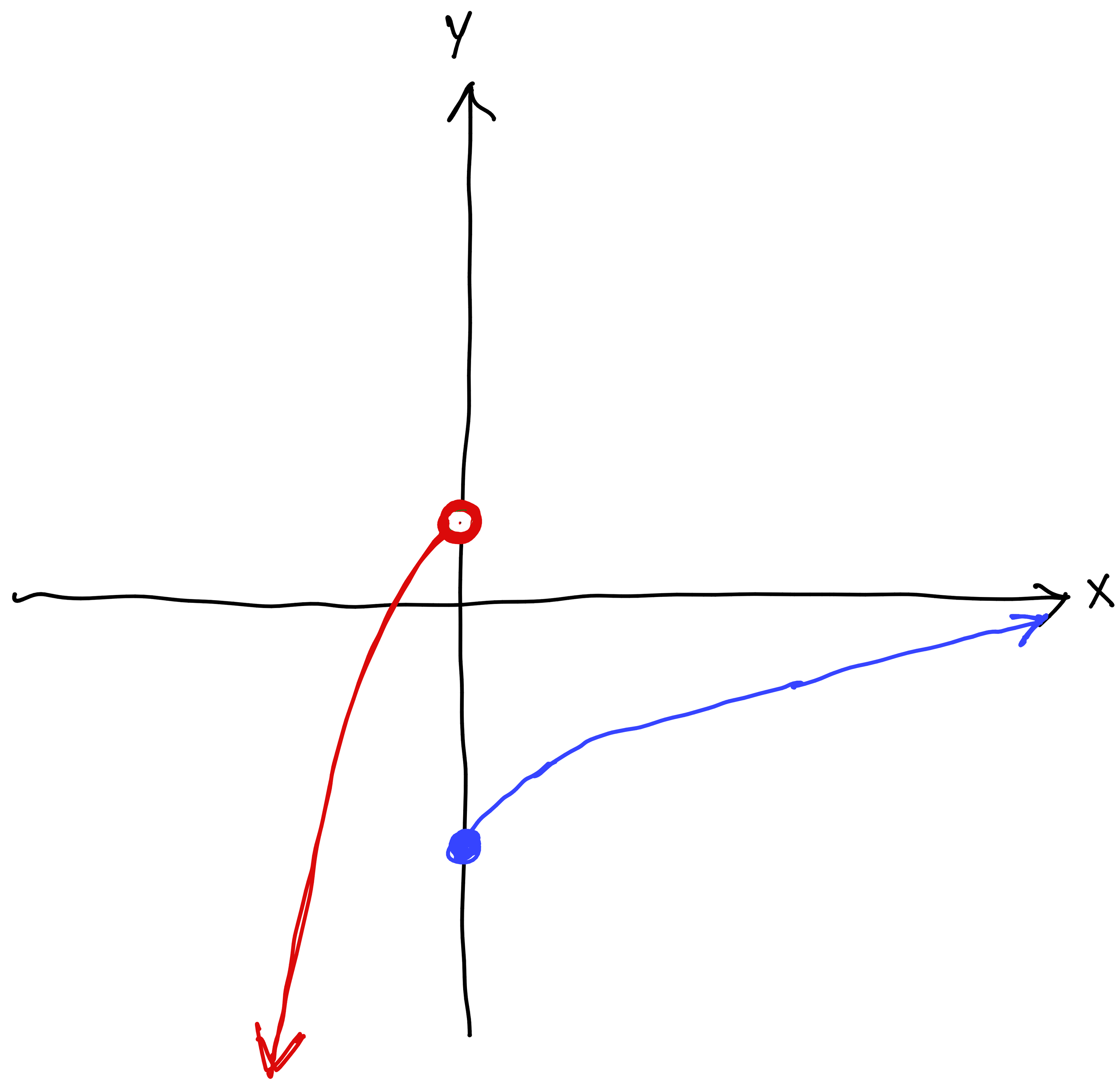
Step 2: horizontal & vertical transf.
$5 \leftarrow$ & $6 \uparrow$

Step 3: y-axis reflection.

$-(x+5)^2-6$ is the same as $-\left((x+5)^2+6\right)$

Piecewise Defined Functions

Ex1: Graph the function $f(x) = \begin{cases} -x^2+1, & \text{if } x<0 \rightarrow x^2 \\ \sqrt{x}-3, & \text{if } x \geq 0 \rightarrow \sqrt{x} \end{cases}$



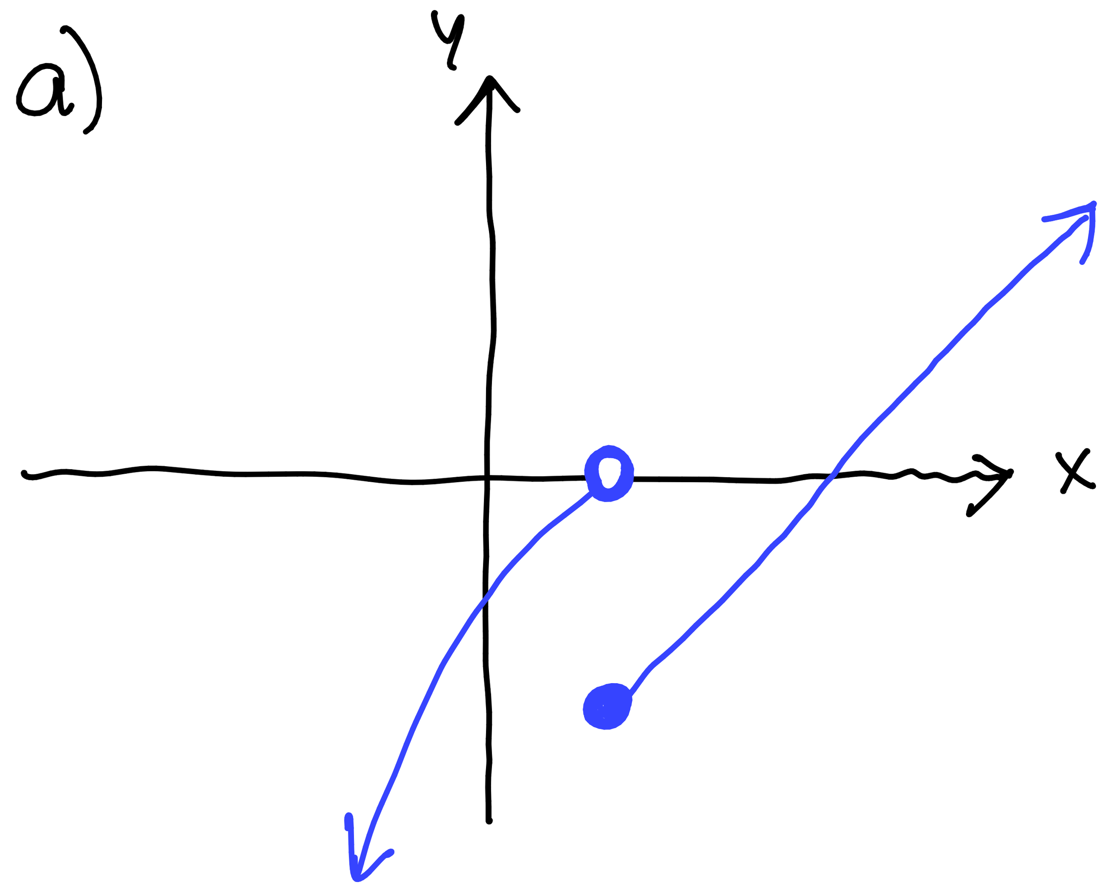
① $-x^2+1 = -(x^2-1)$
$1\downarrow$ & y-axis reflection

② $\sqrt{x}$ & $3\downarrow$

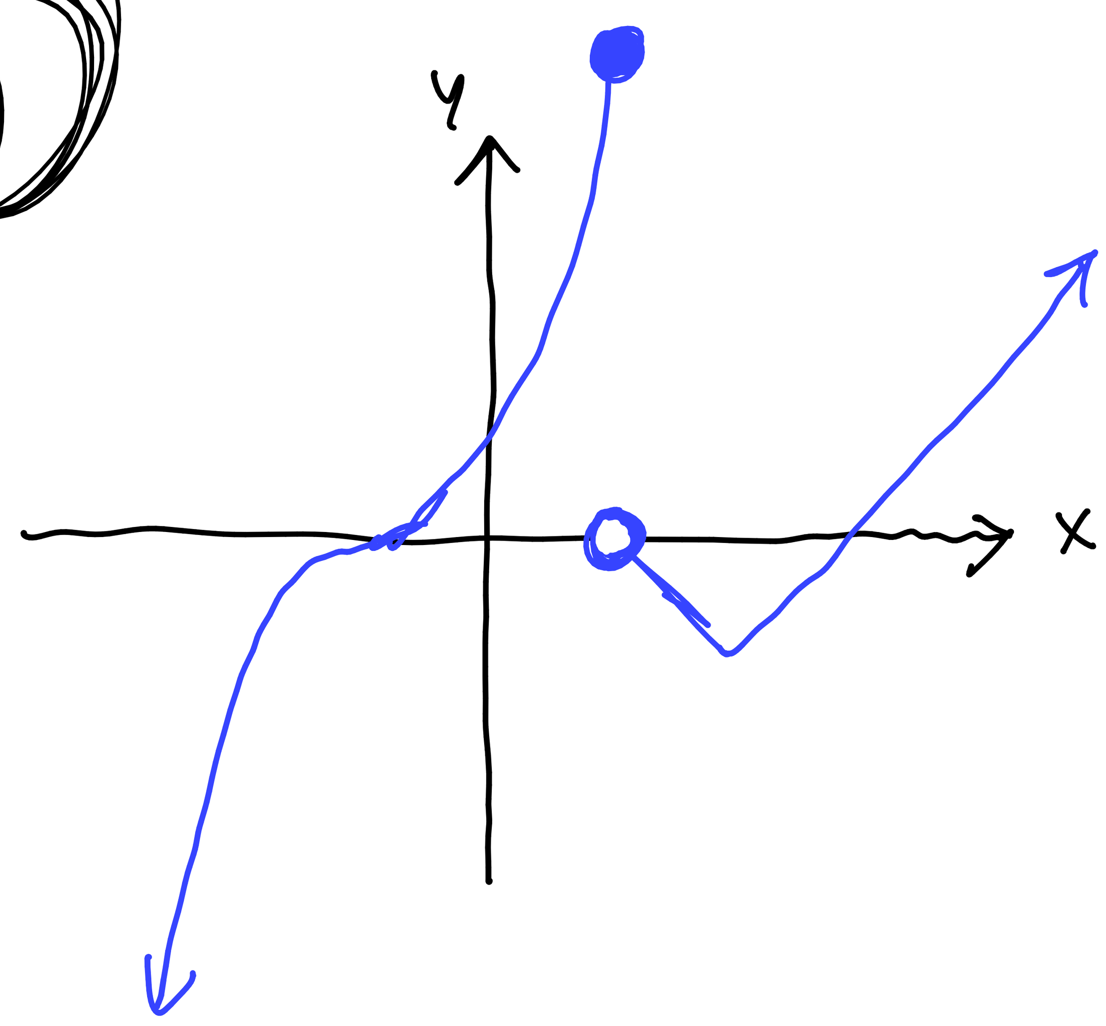
Exercise: Graph the function

$$f(x)=\begin{cases}(x+1)^3, & x\le 1 \quad (*)\\ |x-2|-1, & x>1 \quad (**)\end{cases}$$

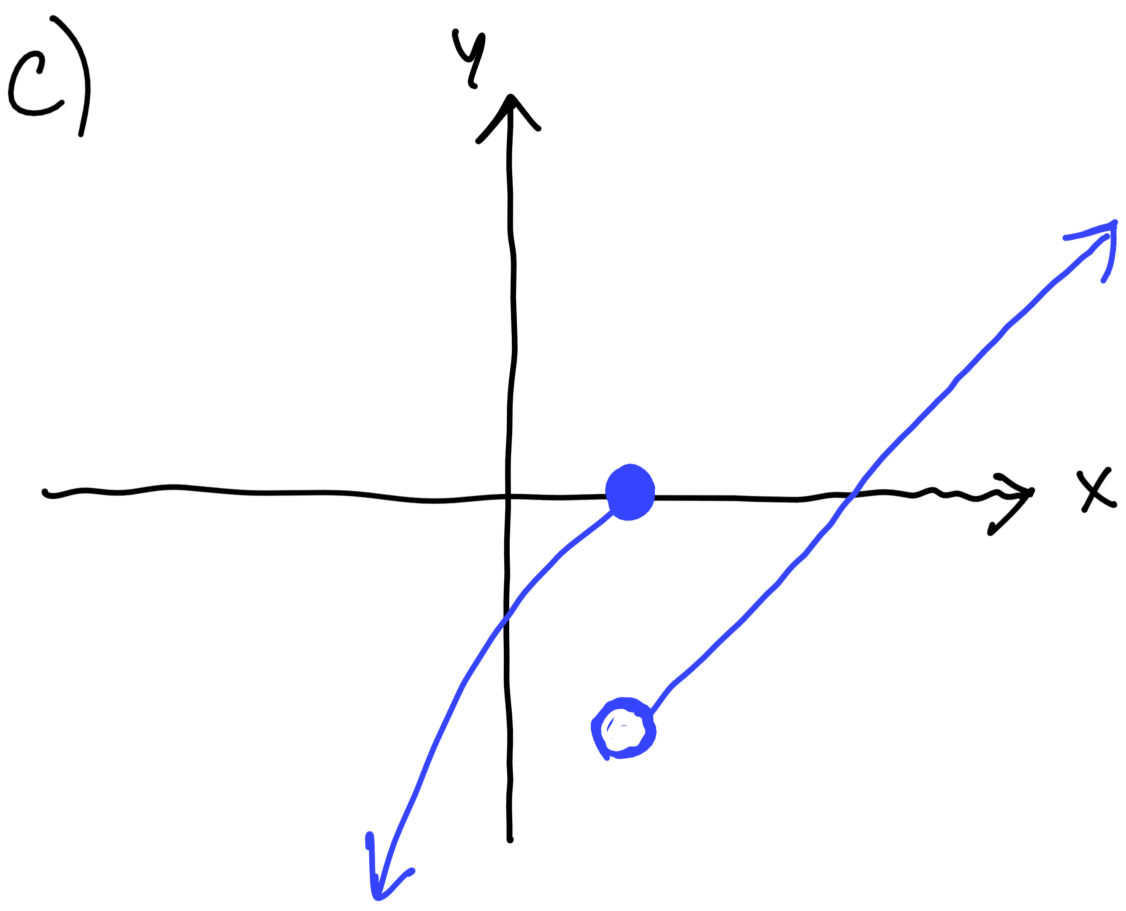
a)



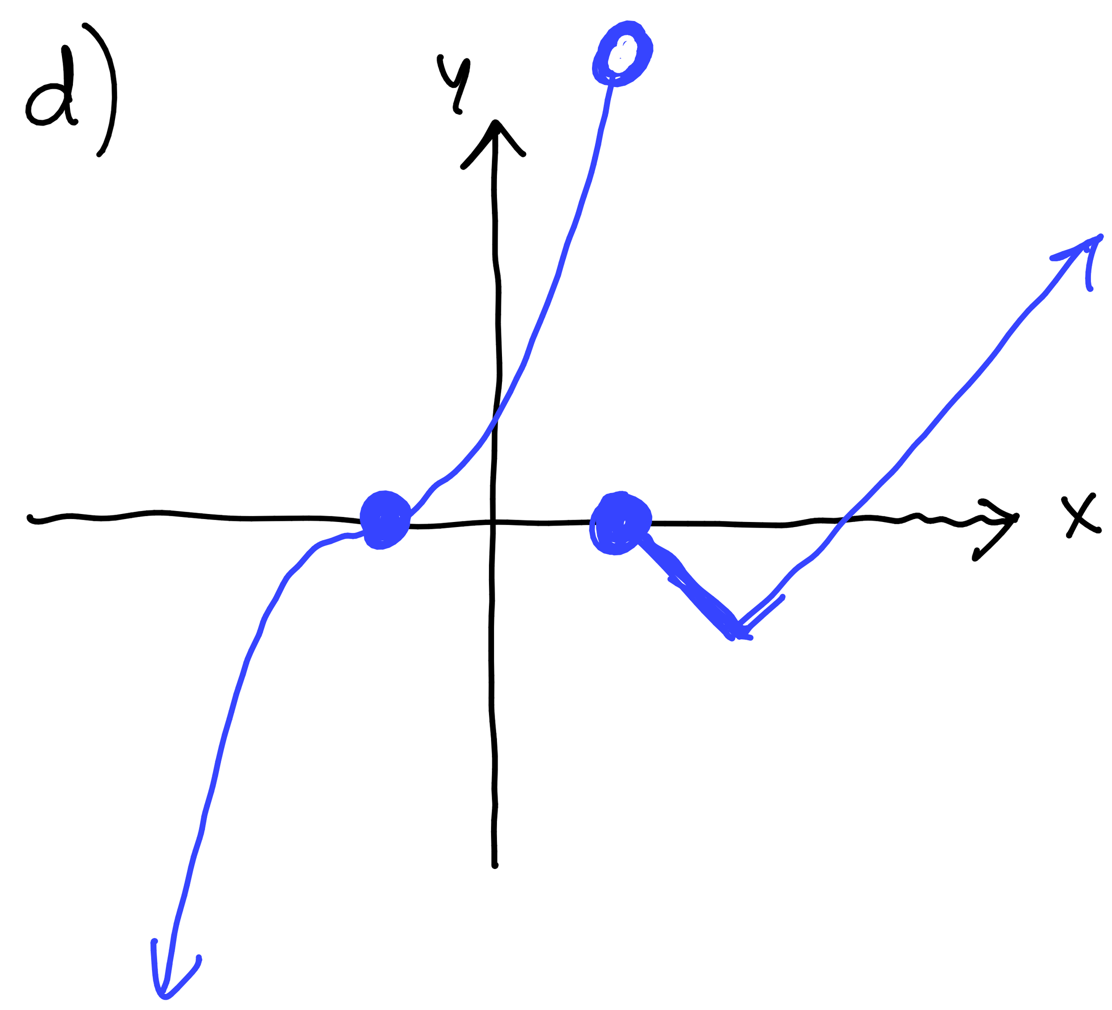
b)



c)



d)



(*) x^3 shifted 1 unit left, and it exists on the left of $x=1$

(**) $|x|$ shifted 2 units right & 1 unit dow, exists right of $x=1$